

# Locality in Nonsequential Quantum Operations

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**Abstract:** We give an example of fulfillment of the condition of locality—no information transfer between certain subsystems—in a tripartite quantum system whose dynamics can not be decomposed (non-sequential dynamics of the system). The three subsystems ( $A$ ,  $B$  and  $C$ ) are designed such that  $C$  interacts simultaneously with both  $A$  and  $B$ , while there is not any interaction between  $A$  and  $B$ . On this basis, we emphasize validity of the condition of locality in a realistic physical situation.

**KEY WORDS:** Quantum locality; quantum subsystems; global unitarity

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## 1. INTRODUCTION

Information transfer is at the heart of information theory, and particularly the issue of locality—i.e. of dividing the universe into subsystems for which information transfer may not be allowed. In a recent article, Schumacher and Westmoreland<sup>(1)</sup> introduced an idea of *dynamical locality* based on a division of a quantum universe into three subsystems, and investigate the consequences of the unitary dynamics of the universe. The condition of locality is described by the requirement of '*no information transfer*' from the subsystem  $B$  to the subsystem  $A$  in the composite system  $A + C + B$ . They conclude that, under a unitary dynamics for  $A + C + B$ , the condition of locality can be satisfied only if interaction between  $A$  and  $C$  precedes interaction between  $C$  and  $B$ , while no interaction between  $A$  and  $B$  is allowed in the system. Nevertheless, the converse does not hold true. Actually, for a unitary dynamics of the composite system, the requirement of locality does not imply sequential interaction as described above, while this may be obtained for a special initial state of  $C$  (a standard initial state denoted  $|0\rangle$ ). This notion exhibits how subtle may be the dynamics of a tripartite quantum system.

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In this paper, we slightly relax and vary some assumptions of the analysis in Ref. 1, and provide a scenario in which *non-sequential* interaction in a tripartite system may still allow the 'no information transfer' between the "remote" (noninteracting) subsystems ( $A$  and  $B$ ). More precisely, we assume the unitary dynamics in the composite system  $A + C + B$ , where  $A$  and  $B$  do *not* mutually interact, while  $C$  *simultaneously* interacts with both  $A$  and  $B$ . We relax the condition of arbitrary initial state of  $C$  by assuming some special (cf. below) initial state  $|0\rangle$ , while assuming the interaction between  $C$  and  $B$  to dominate in the system. As a result, we obtain that, for some time interval, there is not any information transfer between  $A$  and  $B$  in *any direction* possible, while in the limit of the infinite time, there is a possibility of two-directional information transfer between  $A$  and  $B$ . This way the condition of 'no information transfer' may be fulfilled even in case of non-sequential, non-decomposable interaction in the composite system, and may even be extended in both directions. We emphasize subtlety of a tripartite-quantum-system dynamics by illustrating applicability of our scenario to a physical situation typical for certain physical-chemistry situations (e.g. an ion/atom in a solution).

## 2. THE LOCALITY CONDITION

Let us put the notions of Ref. 1 more precisely and introduce the notation.

Schumacher and Westmoreland extend the analysis of Beckman et al<sup>(2)</sup> that refers to the condition of locality in the context of quantum operations, in which one deals with an isolated bipartite system  $A + B$ . Actually, they point out necessity of further "coarse graining" of the isolated system, thus dealing with a tripartite system  $A + C + B$ , for which the condition of dynamical locality is investigated in some generality.

The notion of locality stems: since the state of  $B$  does not alter the state of  $A$ , *no information transfer* is allowed from  $B$  to  $A$ . That is, in order to know the state of  $A$ , only the state of the system  $A + C$  is required to be known—as long as  $A$  and  $C$  are in interaction. The condition of dynamical locality can be introduced in a few, mutually equivalent ways. E.g., nothing we can do to  $B$  affects  $A$  (the "(Locality III)" condition)<sup>(1)</sup>.

Now, the task is to investigate the general conditions for the tripartite system's dynamics that allow the locality condition to be fulfilled. Under the assumption of the unitary dynamics for  $A + C + B$ , the locality condition can be satisfied *only* if the following conditions are fulfilled: (A) the interaction

between  $A$  and  $C$  precedes interaction between  $C$  and  $B$ , and (B)  $A$  and  $B$  are not in mutual interaction. Then, the overall unitary dynamics can be presented as:

$$\hat{U}_{ACB} = (\hat{I}_A \otimes \hat{W}_{CB}) (\hat{V}_{AB} \otimes \hat{I}_B) \quad (1)$$

where both  $\hat{W}$  and  $\hat{V}$  represent the unitary operators of evolution in time; that is, the dynamics is both decomposable and sequential.

On the other hand, one may wonder: if, from the outset it is required the locality to be fulfilled, what can be told about the system dynamics? Interestingly enough, requiring locality to be fulfilled does *not* imply sequential dynamics as distinguished in (1). To this end, the different scenarios are possible, such as<sup>(1)</sup> a *prior interaction* (e.g. measurement) between  $A$  and  $B$  (measurement on  $A$  performed by  $B$ ). In summary: the sequential dynamics (1) guarantees validity of the condition of locality, while the converse does not hold true.

Nevertheless, despite generality of the analysis<sup>(1)</sup>, there might intuitively seem many rooms to be left for certain effects in the tripartite systems not yet obvious or directly predicted by the general analysis in Ref. 1. And this is the subject of the next section, in which we give an example of *simultaneous* (and therefore neither decomposable nor sequential) interaction between  $A$  and  $C$ , and  $C$  and  $B$ , yet without any prior or posterior interaction between  $A$  and  $B$ —while the locality condition is satisfied.

### 3. DISD MODEL

We consider a tripartite system  $A+C+B$  and we slightly departure from the general discussion in Ref. 1 as follows: (i) we assume a special initial state of  $C$ , and (ii) we assume that the interaction between  $C$  and  $B$  dominates in the system.

Let us put these notions in a mathematical form.

First, the *initial* state of the composite system is assumed to read:

$$|\Psi\rangle_{ACB} = \sum_i \alpha_i |i\rangle_A \otimes |0\rangle_C \otimes |\chi\rangle_B \quad (2)$$

bearing obvious notation;  $\sum_i |\alpha_i|^2 = 1$ . The system's Hamiltonian is defined as:

$$\hat{H} = \hat{H}_A + \hat{H}_C + \hat{H}_B + \hat{H}_{AC} + \hat{H}_{CB} \quad (3)$$

where we assume  $\hat{H}_{AB} = 0$ . The crucial assumptions of our model are as follows: (a)  $\hat{H}_{CB}$  dominates in the system, being described by the coupling constant  $\mathcal{C}_1$ , and (b) the initial state  $|0\rangle_C$  is *robust*, relative to the interaction  $\hat{H}_{CB}$ —i.e. the state  $|0\rangle_C$  can not be changed by the interaction  $\hat{H}_{CB}$ :  ${}_C\langle j|\hat{H}_{CB}|0\rangle_C = 0$ ,  $\forall j$  such that  ${}_C\langle j|0\rangle_C = 0$ .

Then, with the aid of the standard (time independent) perturbation theory<sup>(3)</sup>, it can be shown<sup>(4,5)</sup> that the *exact* form of state of the composite system reads:

$$\begin{aligned} |\Psi(t)\rangle_{ACB} = & \sum_i \alpha_i(t) \exp(-it\lambda_{i0}^{(1)}/\hbar) |i\rangle_A \otimes \exp(-it\lambda_0/\hbar) |0\rangle_C \otimes \\ & \otimes \sum_j \beta_j(t) \exp(-it\lambda_{i0j}/\hbar) |j\rangle_B + |O(\epsilon)\rangle_{ACB}. \end{aligned} \quad (4)$$

In eq. (4):  $\lambda_{i0}^{(1)} \equiv {}_{AC} \langle i0|\hat{H}_{CB}|i0\rangle_{AC}$  is a part of the first order correction to the eigenvalues of  $\hat{H}_{CB}$ , while  $\lambda_{i0j}$  involves the higher order corrections to the eigenvalues of  $\hat{H}_{CB}$ , and  $\lambda_0 = {}_C \langle 0|\hat{H}_C|0\rangle_C$ . The quantity  $\epsilon$  (that is implicit in both  $|O(\epsilon)\rangle_{ACB}$  and  $\lambda_{i0j}$ ) is the maximum (or supreme) of the exact corrections to the eigenstates of  $\hat{H}_{CB}$ , its magnitude being proportional to  $\mathcal{C}_2/\mathcal{C}_1$ :  $\mathcal{C}_2$  and  $\mathcal{C}_1$  are the coupling constants of  $\hat{H}_{AC}$  and  $\hat{H}_{CB}$ , respectively. The constants are defined as follows:  $\alpha_i(t) \equiv \alpha_i \exp(-it{}_A \langle i|\hat{H}_A|i\rangle_A)$ , while  $\beta_j(t) \equiv \beta_j \exp[-it(E_{0j}^{(0)} + {}_B \langle j|\hat{H}_E|j\rangle_B)/\hbar]$ ;  $E_{0j}^{(0)}$  is an eigenvalue of  $\hat{H}_{CB}$ , while  $\sum_j |\beta_j|^2 = 1$ . [For more details cf. Refs. 4 and 6.]

As it can be shown<sup>(4)</sup>,  $\| |O(\epsilon)\rangle \| \sim \mathcal{C}_1^{-1}$  as well as  $\lambda \sim \mathcal{C}_1^{-1}$ , where  $\lambda \equiv \sup\{\lambda_{i0j}\}$ . Then, if  $\mathcal{C}_2/\mathcal{C}_1 \rightarrow 0$ , one may state  $\| |O(\epsilon)\rangle \| \rightarrow 0$  and  $\lambda \rightarrow 0$ , which gives *exactly* rise to the lack of correlations in the composite system. For the realistic situations, i.e. when  $\mathcal{C}_2/\mathcal{C}_1 \ll 1$ , while neglecting the small term in (4), one may state the lack of correlations in the time interval  $\tau$  less or of the order of  $\lambda^{-1}$ . For the arbitrarily long time interval much longer than  $\tau$ , there appear the correlations in the composite system. Needless to say, given (4), it is almost trivial to prove the told within the quantum operations formalism (that assumes the use of the "trace" operation).

Physically, for sufficiently strong interaction  $\hat{H}_{CB}$ , the system  $A$  evolves in time approximately *unitarily-like*—as if it were not being an open system—for the period of time of the order of  $\tau$ . And this is the central observation of the model presented.

Here, we want to emphasize: the choice of the initial state as given in (2) need not be physically artificial. First, this choice is adapted to the general analysis<sup>(1)</sup> we start from. Second, the special state  $|0\rangle_C$  is in accordance with the foundations of the decoherence theory<sup>(7)</sup>: the "environment"  $B$  may select such a special state for the (open) system  $C$ , as discussed in Refs. 4 and 6. Therefore, the model (the so-called <sup>(4,5)</sup> DISD model) discussed here seems applicable to the realistic physical situations as discussed briefly in Section 4. Third, the initial state of  $C$  may externally and locally be prepared.

The lack of correlations between  $A$  and  $B$  clearly stems the condition of locality: e.g. reading out a state of  $B$  (of  $A$ ) does not provide any information about the state of  $A$  (of  $B$ )—the "(Locality I)" condition<sup>(1)</sup>. This is even stronger condition of locality than the one investigated in Ref. 1: there is not any information transfer between  $A$  and  $B$ , in any direction possible. Interestingly enough, the interaction in the system is neither decomposable nor sequential. Certainly, for the arbitrarily long time interval (much longer than  $\tau$  defined above), the system  $C$  mediates the information transfer: despite the fact that  $A$  and  $B$  do not mutually interact, the system  $C$  mediates the interaction and provides the correlations of states between  $A$  and  $B$ —cf. the factor  $\lambda_{i0j}$  in (4). Depending on the details in the model (the kind of interactions, the coupling constants and their ratio, the initial states of the subsystems etc.), there might appear variety of the different effects in the system. Our analysis distinguishes such an effect of interest: the dynamics of  $A + C + B$  system can *not* be decomposed, while keeping  $A$  and  $B$  dynamically (and information-theoretically) separated, where interaction between  $A$  and  $B$  is *switched off* in the infinite time,  $t \in (-\infty, \infty)$ , as long as the initial state (2) can be independently—i.e. without interaction between  $A$  and  $B$ —prepared.

#### 4. DISCUSSION AND CONCLUSION

What is the place of our model in the context of the general discussion in Ref. 1?

We use the condition of locality as *additional requirement* to the unitary dynamics of the composite system. As mentioned above (cf. Section 2), then the sequential interaction in the composite system is not required<sup>(1)</sup>. And this is exactly the point at which our considerations fit the general discussion presented in Section 2. Actually, in this context, we relax certain assumptions of the general discussion and it is therefore not for surprise that

we obtain a hopefully interesting result, still extending the notion of locality: not only  $A$  is 'local a system' relative to  $B$ , but the converse holds true. The conditions we have in mind are the points (i) and (ii) in Section 3. Our result reads: for some time interval<sup>2</sup>, the system  $A$  behaves effectively as if it were an isolated system, thus not providing any information for  $B$ , neither  $B$  may provide information transfer to  $A$ . For arbitrarily long time interval, however, there inevitably appear the correlations between  $A$  and  $B$ , thus providing in principle the two-directional information transfer: e.g. reading out the state of  $A$  (of  $B$ ) one may conclude about the state of  $B$  (of  $A$ ).

Physically, the situation described in Section 3 directly refers to the following issues. Originally, the DISD<sup>3</sup> method was developed for the purposes of combating decoherence in the quantum computers hardware<sup>(4)</sup>— a short version may be found in Ref. 5. Nevertheless, the model bears generality, referring to both, suppressing the quantum entanglement in a tripartite quantum system<sup>(6)</sup> as well as to the issue of *avoiding decoherence*—the issue of the decoherence control. As to the former, the model is directly applicable to the following physical situation<sup>(6)</sup>: an atom (or ion—the system  $A$ ) is surrounded by a cage of the solvent molecules (system  $C$ ), which, in turn is in strong interaction with the rest of the solution (the cage's environment—the system  $B$ ). Assuming that the system  $C$  is sufficiently macroscopic—as the general decoherence theory<sup>(7)</sup> stems—the system  $B$  may be able to select<sup>4</sup> a special state  $|0\rangle$  of  $C$ —i.e.  $B$  may decohere  $C$  (which is the origin of the acronym DISD—cf. footnote 3). Due to the strong interaction between  $C$  and  $B$ , this state of  $C$  may remain intact ("robust")<sup>(8)</sup>, i.e. unchanged in time, thus guaranteeing<sup>(4–6)</sup> the unitary-like dynamics for  $A$ . Needless to say, then  $A$  remains effectively decoupled from  $B$  for a time interval of the order of  $\tau$  (cf. above). To this end, the individuality of the atom as well as non-transfer of information (from  $A$  to  $B$  and *vice versa*, and virtually to  $C$ ) represents an example of satisfiability of the locality condition in a realistic physical situation.

Therefore, we conclude, that subtlety of quantum dynamics of the tripartite systems leaves much room for a variety of dynamical effects in the real systems. As we show, slightly relaxing the general assumptions of the dynamical locality model of Schumacher and Westmoreland<sup>(1)</sup>, we are able

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<sup>2</sup>In the limit  $\mathcal{C}_2/\mathcal{C}_1 \rightarrow 0$ , this time interval becomes infinitely long.

<sup>3</sup>Originating from the "decoherence-induced suppression of decoherence"<sup>(4)</sup>.

<sup>4</sup>This situation is mentioned in Section 2: there is a prior interaction between  $A$  and  $B$ : then, the time interval of interest is  $t \in [0, \infty)$ , which challenges validity of (2).

to obtain the dynamical effects not directly predicted within the context of the general discussion. Actually, we provide an example for non-necessity of the sequential interaction *while* the condition of (extended) dynamical locality is satisfied, as well as the possibility of two-directional information flow mediated by the system  $C$  (for arbitrarily long time interval), both within the context of the *same model* of a tripartite system  $A + C + B$ .

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## REFERENCES

1. B. Schumacher and M. D. Westmoreland, *Quantum Information Processing* **4**(1), 13 (2005)
2. D. Beckman, D. Gottesman, M. A. Nielsen, and John Preskill, *Phys. Rev. A* **64**, 052309 (2001)
3. A. Messiah, *Quantum Mechanics* (North Holland Publishing Company, Amsterdam, 1976)
4. M. Dugić, *Quantum Computers & Computing* **1**, 102 (2000)
5. M. Dugić, e-print arXive quant-ph/0001009
6. M. Dugić, *Europhys. Lett.* **60**, 7 (2002)
7. D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu and H. D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996)
8. W. H. Zurek, *Prog. Theor. Phys.* **89**, 281 (1993)